SOLUTION OF EXERCISE # 1.4

Exercise # 1.4

Q.1: Form quadratic equations with the following given numbers as its roots.

(i) $2, -3$

$S = 2 + (-3)$

$S = 2 - 3 \Rightarrow S = -1$

$P = (2)(-3)$

$P = -6$

$$x^2 - Sx + P = 0$$

$$x^2 - (-1)x + (-6) = 0 \Rightarrow \boxed{x^2 + x - 6 = 0}$$

(ii) $3 + i, \; 3 - i$ (IIA-2017)

$S = 3 + \cancel{i} + 3 - \cancel{i}$

$S = 6$

$P = (3 + i)(3 - i)$

$P = 9 - \cancel{3i} + \cancel{3i} - i^2$

$P = 9 - (-1)$

$P = 9 + 1 \Rightarrow P = 10$

$$x^2 - Sx + P = 0 \Rightarrow \boxed{x^2 - 6x + 10 = 0}$$

(iii) $2 + \sqrt{3}, 2 - \sqrt{3}$

$S = 2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}}$

$S = 4$

$P = (2 + \sqrt{3})(2 - \sqrt{3})$

$P = (2)^2 - (\sqrt{3})^2$

$P = 4 - 3 \Rightarrow P = 1$

$$x^2 - Sx + P = 0 \Rightarrow \boxed{x^2 - 4x + 1 = 0}$$

(iv) $-3 + \sqrt{5}, \; -3 - \sqrt{5}$

$S = (-3 + \sqrt{5}) + (-3 - \sqrt{5})$

$S = -3 + \cancel{\sqrt{5}} + -3 - \cancel{\sqrt{5}}$

$S = -6$

$P = (-3 + \sqrt{5})(-3 - \sqrt{5})$

$P = (-3)^2 - (\sqrt{5})^2$

$P = 9 - 5 \Rightarrow P = 4$

$$x^2 - Sx + P = 0$$

$$x^2 - (-6)x + 4 = 0 \Rightarrow \boxed{x^2 + 6x + 4 = 0}$$

SOLUTION OF EXERCISE # 1.4

(v) $4+5i\ ,\ 4-5i$

$S=4+\cancel{5i}+4-\cancel{5i}$	$P=(4+5i)(4-5i)$
$S=83$	$P=(4)^2-(5i)^2$
	$P=16-25(-1)$
	$P=16+25=41$

$x^2-Sx+P=0 \quad \Rightarrow \quad \boxed{x^2-8x+41=0}$

Q.2: Find the quadratic equations with roots

(i) Equal numerically but opposite in sign to those of the roots of the equation,
$3x^2+5x-7=0$

Sol. Let α, β are the roots of the given equation.

$$3x^2+5x-7=0$$

Here $a=3, \qquad b=5, \qquad c=-7$

Sum of Roots $=\dfrac{-b}{a}$	Products of Roots $=\dfrac{c}{a}$
$\alpha+\beta=\dfrac{-5}{3}$	$\alpha\beta=\dfrac{-7}{3}$

So, $-\alpha, -\beta$ are the roots of require equation.

$S=-\alpha+(-\beta)$	$P=(-\alpha)(-\beta)$
$S=-\alpha-\beta$	$P=\alpha\beta$
$S=-(\alpha+\beta)$	$P=-\dfrac{7}{3}$
$S=-\left(-\dfrac{5}{3}\right)=\dfrac{5}{3}$	

$$x^2-Sx+P=0$$

$$x^2-\frac{5}{3}x+\left(\frac{-7}{3}\right)=0$$

Multipling both sides by 3, we get

$$\Rightarrow \quad \boxed{3x^2-5x-7=0}$$

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(ii) Twice the roots of the equation $5x^2+3x+2=0$

Sol. Let α, β are the roots of the given equation.

$$5x^2 + 3x + 2 = 0$$

Here $a = 5$, $b = 3$, $c = 2$

Sum of Roots $= \frac{-b}{a}$	Products of Roots $= \frac{c}{a}$
$\alpha + \beta = -\frac{3}{5}$	$\alpha\beta = \frac{2}{5}$

So, 2α, 2β are the roots of the require equation.

$S = 2\alpha + 2\beta$	$P = (2\alpha)(2\beta)$
$S = 2(\alpha + \beta)$	$P = 4\alpha\beta$
$S = 2\left(-\frac{3}{5}\right)$	$P = 4\left(\frac{2}{5}\right)$
$S = -\frac{6}{5}$	$P = \frac{8}{5}$

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{-6}{5}\right)x + \frac{8}{5} = 0$$

Multipling both sides by 5, we get:

$$\Rightarrow \quad \boxed{5x^2 + 6x + 8 = 0}$$

Q.3: Form the quadratic equation whose roots are less by '1' then those of $3x^2 - 4x - 1 = 0$

Sol. Let α, β are the roots of the given equation.

$$3x^2 - 4x - 1 = 0$$

Here $a = 3$, $b = -4$, $c = -1$

Sum of Roots $= \frac{-b}{a}$	Products of Roots $= \frac{c}{a}$
$\alpha + \beta = -\left(-\frac{4}{3}\right) = \frac{4}{3}$	$\alpha\beta = \frac{-1}{3}$

SOLUTION OF EXERCISE # 1.4

So, $\alpha-1, \beta-1$ are the roots of the require equation.

$$S=(\alpha-1)+(\beta-1)$$
$$S=\alpha-1+\beta-1$$
$$S=\alpha+\beta-2$$
$$S=\frac{4}{3}-2$$
$$S=\frac{4-6}{3}=\frac{-2}{3}$$

$$P=(\alpha-1)(\beta-1)$$
$$P=\alpha\beta-\alpha-\beta+1$$
$$P=\alpha\beta-(\alpha+\beta)+1$$
$$P=-\frac{1}{3}-\frac{4}{3}+1$$
$$P=\frac{-1-4+3}{3}=\frac{-2}{3}$$

$$x^2-Sx+P=0$$
$$x^2-\left(\frac{-2}{3}\right)x-\frac{2}{3}=0$$

Multipling both sides by 3, we get

$$\Rightarrow \quad \boxed{3x^2+2x-2=0}$$

Q.4: Form the quadratic equation whose roots are the square of the roots of the equation $2x^2-3x-5=0$

Sol. Let α, β are the roots of the given equation.

$$2x^2-3x-5=0$$

Here $a=2, \quad b=-3, \quad c=-5$

$$\text{Sum of Roots}=\frac{-b}{a}$$
$$\alpha+\beta=-\left(-\frac{3}{2}\right)=\frac{3}{2}$$

$$\text{Products of Roots}=\frac{c}{a}$$
$$\alpha\beta=\frac{-5}{2}$$

So, α^2, β^2 are the roots of the require equation.

$$S=\alpha^2+\beta^2$$
$$S=(\alpha+\beta)^2-2\alpha\beta$$
$$S=\left(\frac{3}{2}\right)^2-2\left(-\frac{5}{2}\right)$$

$$P=\alpha^2\beta^2$$
$$P=(\alpha\beta)^2$$
$$P=\left(-\frac{5}{2}\right)^2$$

SOLUTION OF EXERCISE # 1.4

$S = \frac{9}{4} + 5$

$S = \frac{9+20}{4} \Rightarrow \boxed{S = \frac{29}{4}}$

$\boxed{P = \frac{25}{4}}$

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{29}{4}x + \frac{25}{4} = 0$$

Multipling both sides by 4, we get

$$\Rightarrow \boxed{4x^2 - 29x + 25 = 0}$$

Q.5: Find the quadratic equation whose roots are reciprocal of the roots of the equation $px^2 - qx + r = 0$

Sol. Let α, β are the roots of given equation.

$$px^2 - qx + r = 0$$

Here $a = p$, $b = -q$, $c = r$

Sum of Roots $= \frac{-b}{a}$

$\alpha + \beta = -\left(-\frac{q}{p}\right) = \frac{q}{p}$

Products of Roots $= \frac{c}{a}$

$\alpha\beta = \frac{r}{p}$

So, $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of the require equation.

$S = \frac{1}{\alpha} + \frac{1}{\beta}$

$S = \frac{\beta + \alpha}{\alpha\beta}$

$S = \frac{\alpha + \beta}{\alpha\beta}$

$S = \frac{\cancel{q}/\cancel{p}}{r/\cancel{p}} = \frac{q}{r}$

$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$

$P = \frac{1}{\alpha\beta}$

$P = \frac{1}{\frac{r}{p}}$

$P = \frac{p}{r}$

SOLUTION OF EXERCISE # 1.4

$$x^2 - Sx + P = 0 \Rightarrow x^2 - \frac{q}{r}x + \frac{p}{r} = 0$$

Multipling both sides by 'r', we get: $\Rightarrow \boxed{rx^2 - qx + p = 0}$

Q.6: If α, β are the roots of the equation $x^2 - 4x + 2 = 0$, find the equations whose roots are:

(i) α^2, β^2

Sol. As α, β are the roots of the given equation.

$$x^2 - 4x + 2 = 0$$

Here $a = 1$, $b = -4$, $c = 2$

Sum of Roots $= \frac{-b}{a}$

$$\alpha + \beta = -\left(-\frac{4}{1}\right) = 4$$

Products of Roots $= \frac{c}{a}$

$$\alpha\beta = \frac{2}{1} = 2$$

Given that, α^2, β^2 are the roots of the require equation.

$$S = \alpha^2 + \beta^2$$
$$S = (\alpha + \beta)^2 - 2\alpha\beta$$
$$S = (4)^2 - 2(2)$$
$$S = 16 - 4 = 12$$

$$P = \alpha^2\beta^2$$
$$P = (\alpha\beta)^2$$
$$P = (2)^2$$
$$P = 4$$

$$x^2 - Sx + P = 0 \Rightarrow \boxed{x^2 - 12x + 4 = 0}$$

(ii) α^3, β^3

Sol. Given that, α^3, β^3 are the roots of the require equation.

$$S = \alpha^3 + \beta^3$$
$$S = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$
$$S = (4)^3 - 3(2)(4)$$
$$S = 64 - 24 = 40$$

$$P = \alpha^3\beta^3$$
$$P = (\alpha\beta)^3$$
$$P = (2)^3$$
$$P = 8$$

$$x^2 - Sx + P = 0 \Rightarrow \boxed{x^2 - 40x + 8 = 0}$$

SOLUTION OF EXERCISE # 1.4

(iii) $\alpha+\frac{1}{\alpha}, \beta+\frac{1}{\beta}$

Sol. Given that, $\alpha+\frac{1}{\alpha}, \beta+\frac{1}{\beta}$ are the roots of the require equation.

$$S=\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}$$
$$S=\alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}$$
$$S=(\alpha+\beta)+\frac{\alpha+\beta}{\alpha\beta}$$
$$S=4+\frac{4}{2}$$
$$S=4+2$$
$$S=6$$

$$P=\left(\alpha+\frac{1}{\alpha}\right)\left(\beta+\frac{1}{\beta}\right)$$
$$P=\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}$$
$$P=\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha\beta}+\frac{1}{\alpha\beta}$$
$$=\alpha\beta+\frac{1}{\alpha\beta}+\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$
$$=2+\frac{1}{2}+\frac{(4)^2-2(2)}{2}$$
$$=2+\frac{1}{2}+\frac{16-4}{2}=2+\frac{1}{2}+\frac{12}{2}$$
$$=\frac{4+1+12}{2}=\frac{17}{2}$$

$$x^2-Sx+P=0$$
$$x^2-6x+\frac{17}{2}=0$$

Multipling both sides by 2, we get

$$\Rightarrow \boxed{2x^2-12x+17=0}$$

(iv) $\alpha+2, \beta+2$

Sol. Given that, $\alpha+2, \beta+2$ are the roots of the require equation.

$$S=\alpha+2+\beta+2$$
$$S=\alpha+\beta+4$$

$$P=(\alpha+2)(\beta+2)$$

SOLUTION OF EXERCISE # 1.4

$S = 4 + 4$

$S = 8$

$P = \alpha\beta + 2\alpha + 2\beta + 4$

$P = \alpha\beta + 2(\alpha + \beta) + 4$

$P = 2 + 2(4) + 4 = 14$

$x^2 - Sx + P = 0 \quad \Rightarrow \quad \boxed{x^2 - 8x + 14 = 0}$

Q.7: If α, β are roots of $ax^2 + bx + c = 0$, form an equation whose roots are:

(i) $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

Sol. $S = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\frac{c}{a}}$$

$$S = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}$$

$$S = \frac{b^2 - 2ac}{a^2} \times \frac{a}{c}$$

$$S = \frac{b^2 - 2ac}{ac}$$

$$P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$$

(ii) $\dfrac{\alpha^2}{\beta}, \dfrac{\beta^2}{\alpha}$

Sol. $S = \dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}$

$$S = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$S = \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$$

$$S = \frac{\left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)}$$

$$S = \left(-\frac{b^3}{a^3} + \frac{3bc}{a^2}\right) \times \frac{a}{c}$$

$$S = \frac{-b^3 + 3abc}{a^3} \times \frac{a}{c}$$

$$S = \frac{-b^3 + 3abc}{a^2c}$$

$$P = \left(\frac{\alpha^2}{\beta}\right)\left(\frac{\beta^2}{\alpha}\right) = \alpha\beta = \frac{c}{a}$$

SOLUTION OF EXERCISE # 1.4

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{(b^2 - 2ac)}{ac}x + 1 = 0$$

Multipling by ac, we get

$$\boxed{acx^2 - (b^2 - 2ac)x + ac = 0}$$

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{(-b^3 + 3abc)}{a^2c}x + \frac{c}{a} = 0$$

Multipling by a^2c, we get

$$\boxed{a^2cx^2 + (b^3 - 3abc)x + ac^2 = 0}$$

(iii) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$

Sol. $S = \frac{\alpha+1}{\alpha} + \frac{\beta+1}{\beta}$

$$= \frac{\beta(\alpha+1) + \alpha(\beta+1)}{\alpha\beta}$$

$$= \frac{\alpha\beta + \beta + \alpha\beta + \alpha}{\alpha\beta}$$

$$= \frac{\alpha + \beta + 2\alpha\beta}{\alpha\beta} = \frac{\frac{-b}{a} + 2\frac{c}{a}}{\frac{c}{a}}$$

$$= \frac{-b + 2c}{a} \times \frac{a}{c}$$

$$= \frac{-b + 2c}{c} = \frac{2c - b}{c}$$

$$p = \left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\beta+1}{\beta}\right)$$

$$P = \frac{\alpha\beta + \alpha + \beta + 1}{\alpha\beta}$$

$$P = \frac{\alpha\beta + (\alpha + \beta) + 1}{\alpha\beta}$$

$$P = \frac{\frac{c}{a} + \left(-\frac{b}{a}\right) + 1}{c/a}$$

$$P = \left(\frac{c - b + a}{a}\right) \times \frac{a}{c}$$

$$P = \frac{a - b + c}{c}$$

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{2c - b}{c}\right)x + \left(\frac{a - b + c}{c}\right) = 0$$

Multipling by c, we get: $\boxed{cx^2 - (2c - b)x + (a - b + c) = 0}$